A New Approach in Understanding Growth and Decay of the Sunspots

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ABSTRACT

From the previous study (Hiremath 2009b; Hiremath 2010), on the genesis of solar cycle and activity phenomena, it is understood that sunspots are formed at different depths by superposition of Alfven wave perturbations of a strong toroidal field structure in the convective envelope and after attaining a critical strength, due to buoyancy, raise toward the surface along the rotational isocontours that have positive (0.7-0.935 $R_{\odot}$) and negative (0.935-1.0 $R_{\odot}$) rotational gradients. Owing to physical conditions in these two rotational gradients, from the equation of magnetic induction, sunspot's area growth and decay problem is solved separately. It is found that rate of growth of sunspot's area during its evolution at different depths is function of steady and fluctuating parts of Lorentzian force of the ambient medium, fluctuations in meridional flow velocity, radial variation of rotational gradient and $cot(\vartheta)$ (where ϑ is co-latitude). While rate of decay of sunspot's area at different depths during its evolution mainly depends upon magnetic diffusivity, rotational gradient and $sin^2(\vartheta)$. Gist of this study is that growth and decay of area of the sunspot mainly depends upon whether sunspot is originated in the region of either positive or negative rotational gradient.

On the surface, as fluctuating Lorentz forces and meridional flow velocity during sunspots' evolution are considerably negligible compared to steady parts, analytical solution for growth of sunspot area A is $A(t) = A_0 e^{(U_0 cot\vartheta)t/2}$ (where A_0 is area of the sunspot during its' initial appearance and U_0 is steady part of meridional flow velocity on the surface, ϑ is co-latitude and t is a time variable). Similarly analytical solution for decay of sunspot's area on the surface follows the relation $A(t) = C_1 e^{-(\frac{\Omega_0^2 R_{\odot}^2 sin^2\theta}{\eta})t} + C_2$ (where C_1 and C_2 are the integrational

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constants, $R_\odot$ is radius of the sun, Ω_0 steady part of angular velocity and η is the magnetic diffusivity). For different latitudes and life spans of the sunspots on the surface during their evolutionary history, both the analytically derived theoretical area growth and decay curves match reasonably well with the observed area growth and decay curves.

1. Introduction

Since discovery of the sunspots by Galileo, genesis of their 22 year cyclic activity in general and, their formation and decay during their evolutionary stages in particular still remain a mystery. The study of sunspots' origin, formation and decay is important owing to the observed fact that variation of sunspot occurrence activity is related with the solar irradiance that in turn affects the earth's environment and the climate (Prabhakaran Nayar *et. al.* 2002; Hiremath and Mandi 2004; Soon 2005; Badruddin, Singh and Singh 2006; Perry 2007; Feymann 2007; Tiwari and Ramesh 2007 and references there in; Scafetta and West 2008 Komitov 2009, Hiremath 2009b and references there in).

Present general consensus is that the sunspots originate below the solar surface due to an unknown dynamo mechanism. Due to very high conductivity of the solar plasma and assuming that raising flux tube does not acquires extra flux from the ambient medium, sunspots isorotate with the internal plasma and due to buoyancy raise toward the surface along the path of rotational isocontours. This implies that sunspots are very good tracers of the internal dynamics and structure of the solar interior. Hence if the sunspots that have first and second days appearance on the surface, and if one computes their initial rotation rates, then one can infer rotation rate of the internal solar plasma where the sunspots' foot points are anchored. Recent studies (Javaraiah and Gokhale 1997; Javaraiah 2001; Hiremath 2002; Sivaraman *et. al.* 2003) show that variation of initial rotation rates obtained from the daily motion of sunspot groups with respect to their life spans on the surface is almost similar to the radial variation of the internal rotation profile of the solar plasma.

From Hiremath's (2002) paper, results are reproduced in Fig 1 that illustrates a comparison between the variation of initial rotation rates of the sunspot groups for different life spans and radial variation of internal rotation profile as inferred (Antia, Basu and Chitre 1998) from the helioseismology. Note the striking similarity between these two profiles. In order to reach closer to the reality of the physics of convection zone and dynamics of the flux tubes, in the same study, the rate of change of initial rotation rates of the sunspot groups (that represent the acceleration or deceleration of the flux tubes in the ambient plasma) are compared (the Fig 5(b) of Hiremath (2002) with the radial profile of gradient of rotation

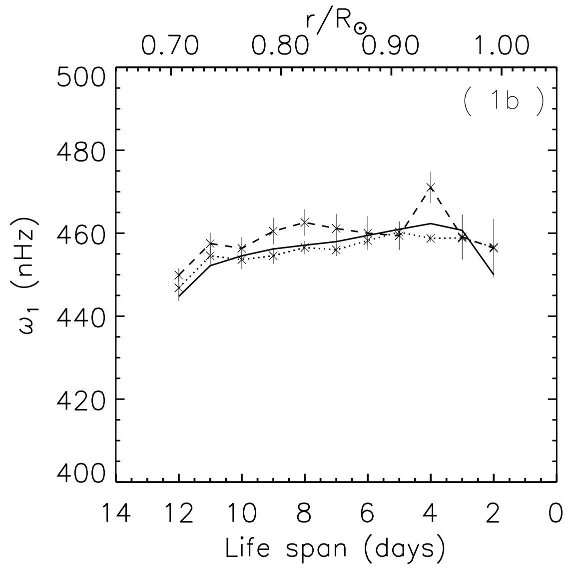

Fig. 1.— The dashed and the dotted curves are the variation of initial rotation rates of the sunspot groups with respect to their life spans (Hiremath 2002). The continuous curve is the radial variation of the internal rotation as inferred from the helioseismologyi (Antia, Basu and Chitre 1998).

that is computed from the radial variation of rotation of the plasma inferred from the helioseismology). Again we get a striking similarity between these profiles. To conclude from that study, for different life spans, initial sunspot dynamics over the surface represents the internal dynamics in different layers of the convection zone. For example initial anchoring of a flux tube whose life span is 10 days is near base of the convection zone and initial anchoring of a flux tube whose life span is 5 days is in the middle of the convective envelope.

Observations show that there are three important stages in the sunspot's evolutionary history : (i) a well developed sunspot (that consists of umbra and penumbra) is formed due to coalescing of the emerging flux regions, (ii) once stabilized sunspot is formed, its area increases and reach the maximum value and, (iii) decay of the sunspot from it's maximum area to minimum area and ultimately disintegrating into smaller active regions and diffusion of the flux on the surface.

As for the first and last stages, there are many studies that explain the formation and decay parts of the sunspot's evolutionary history. The first stage is supposed to be due to convective collapse, a kind of instability that has been invoked to explain the kilo gauss fields on the surface (Parker 1978; Spruit 1979; Hasan 1985). Once flux element is formed, different

adjacent flux elements coalesce and sunspot is formed. Owing to their strong magnetic field structure, sunspots inhibit the ambient convection resulting in reduction of temperature and density. Ultimately lower density of the flux tube results in raising (due to buoyancy) from the convection zone to the surface. Contrary to this conventional view, Parker (1992) has proposed that sunspots are basically formed due to coalescence of magnetic elements by the vortices. According to him, flux tubes are surrounded by vortex flows that attract other vortices leading to coalescence of different flux elements. On the other hand Meyer *et. al.* (1974), have different view on the formation of the flux tubes. According to them a strong converging flow is necessary to form the sunspots. That means sunspots might be formed at the boundary of the convective cells, with an outflow at the surface and an inflow in the deeper layers. Where as Hiremath (2009b; 2010), by updating Alfven's (1943) seminal idea of sunspot formation, came to the conclusion that sunspots are formed due to superposition of Alfven wave perturbations of the underlying steady part of large scale toroidal magnetic field structure and travel along isorotational contours in order to reach at the proper activity belt on the surface.

There are many studies on the decaying phase of the sunspot. Cowling (1946) was the first person to investigate the decay part of the sunspot area. Bumba (1963) obtained a linear decay law for the recurrent spot groups and exponential decay law for the non-recurrent spot groups. Where as some of the previous studies (Petrovay and Moreno-Insertis 1997; Petrovay and van Driel-Gesztelyi 1997) indicate the quadratic decay (*i.e.*, sunspot area as quadratic function of time) and other studies (Solanki 2003 and references there in) indicate the linear decay law. Moreno-Insertis and Vazquez (1988) and Martinez Pillet, Moreno-Insertis and Vazquez (1993) conclude that the present sunspot data do not allow any distinction between either linear or quadratic decay law. To add to these decay laws, log-normal distribution (Martinez Pillet, Moreno-Insertis and Vazquez 1993) also fit the decay of umbrae.

There are following theoretical studies to understand the sunspot decay. First theoretical study in supporting the results of linear decay laws is by Zwan and Gokhale (1972). Such a linear decay law suggests that flux loss takes place everywhere within the spot irrespective of their different sizes. Zwan and Gokhale (1972) assumed a current sheet around the sunspot and turbulent diffusion inside the tube. In this case Ohmic diffusion dictates the decay of the current sheet and hence as spot decays to smaller area, thickness of the current sheet reduces. In fact such current sheets around the sunspots have been observed by Solanki, Rueedi and Livingston (1992). In contrast, Simon and Leighton (1964) and Schmidt (1968) propose that the sunspots are decayed by the erosion of the sunspot boundary which implies that dA/dt is proportional to $A^{1/2}$, where A is area of spot. Supporting the erosion model, Petrovay and Moreno-Insertis (1997) proposed that turbulent diffusivity depends strongly on the field strength. Their model predicts the quadratic decay and spontaneous current

sheet around the sunspot.

Though there are many studies on the first and last phases of the sunspot evolution, the second stage of a sunspot, viz., physics of a growth phase, during it's life time is not understood. Moreover, it is not clear whether all the three phases in a sunspot's life time remain same or different over the whole solar cycle. That means: is there any year to year variations in the area gradients (rate of change of area dA/dt with respect to time, where A(t) is time dependent area of the sunspots and t is time variable) of the sunspots during it's increasing (second phase) and decaying (last phase)? Is there any connection between the evolutionary history of the sunspots and underlying deeper dynamics or this phenomenon is simply due to surface convection. Some of these important issues are addressed in this study.

As for year to year changes in gradient of sunspots' area, for the year 1955-1965 and for different life spans, Hiremath (2005, with summer student Mr. Subba Rao), computed both growth dA_1/dt and the decay dA_2/dt rates of the sunspots and came to the following conclusions. For the same life span, to reach their maximum areas sunspots take different times as cycle progresses. That is, in the beginning of the cycle, area-age curves are nearly gaussian and as cycle progresses area-age curve follow the simple linear decay law. Further they conclude: (i) during the beginning of the solar cycle, sunspots' *rate of growth* and *rate of decay* are larger compared at end of the solar cycle, (ii) in the beginning of the solar cycle, in order to reach their maximum areas, sunspots increase their area at a rate of $\sim$ 100 mh (millionths hemisphere)/day where as at the end of solar cycle sunspots increase their area at the rate of $\sim$ 50 mh/day and, (iii) sunspots decay faster ($\sim$ 75 mh/day) in the beginning of the solar cycle compared to the end of the solar cycle ($\sim$ 25 mh/day). The last conclusion is similar to the conclusion from the recent study (Hathaway and Choudhary 2008) on decay of area of the sunspot groups. Active regions are centers of solar activity ranging from flares to CMEs. They are believed to be locations where magnetic flux bundles erupt from deep interior in the convection zone to emerge at solar surface in the form of sunspots due to magnetic buoyancy. Further the complexity of sunspot groups plays an important role in determining the active regions (Zirin 1988). The difference in the total energy output between a solar minimum and maximum, indirectly associated with the sunspots, is about 0.1%. Even this small energy changes in the sun's output over 11-year solar cycle can intensify wind and rainfall activities and therefore has a major impact on global weather and climatic patterns in the Earth's climatic parameters. Therefore it is useful to investigate how the sunspot groups themselves eventually grow and decay. The growth and decay of sunspot groups also play an important role in the day to day irradiance variations (Wilson 1981).

If decay of sunspots were purely by ohmic dissipation, sunspots would have lifetimes

of about 300 years by considering their size and photospheric conductivity (Cowling 1946). However, the sunspots have shorter life span of $\sim$ weeks for non-recurrent spot groups and $\sim$ months for recurrent spot groups. How to reconcile these observed phenomena, viz., three phases of formation, growth and decay of the sunspots. In the recent study (Hiremath 2009b; Hiremath 2010), it is proposed that sunspots are formed by the superposition of many Alfven wave perturbations of the embedded toroidal magnetic field structure. Once sunspots are formed, due to buoyancy, at different depths in the convective envelope raise along isorotational contours and reach the surface at different latitudes. One can notice from Fig 1 that the internal rotational profile (continuous curve), as inferred from helioseismology (Antia, Basu and Chitre 1998), has two rotational gradients, *viz.,* a positive rotational gradient from base of the convective envelope to $0.935R_{\odot}$ and a negative rotational gradient from $0.935R_{\odot}$ to $1.0R_{\odot}$.

From the magnetic induction equation, it is proposed in this study that growth and decay of either sunspots' area or magnetic flux is due to interplay of both convective source term (that in turn depends mainly upon fluctuations in the positive rotational gradient and convection) and sink term (that in turn depends upon fluctuations in negative rotational gradient, magnetic eddy diffusivity and radiation effects near the surface). That means sunspots that are formed in the region of positive rotational gradient, while raising toward the surface, accumulate magnetic flux from the ambient magnetic turbulent medium and reduction of magnetic flux in the region of negative rotational gradient. The net magnetic flux of the sunspot that is formed in the region of positive rotational gradient in the convective envelope while raising it's anchoring feet and reaching toward $0.935R_{\odot}$, should increase and, magnetic flux should decrease as flux tubes' anchoring feet lifts from $0.935R_{\odot}$ to $1.0R_{\odot}$. On the other hand, the sunspots that are formed in the region of negative rotational gradient while raising toward the surface mainly experience decay phase only. These reasonable ideas will be clear in the following section. In order to understand and test these ideas on growth and decay phases of the sunspots, magnetic induction equation is solved by considering the source and the sink terms separately. In section 2, formulation of the equations are presented. With reasonable approximations, analytical solution of magnetic induction equation for the growth of area is presented in section 3 and solution for decay part is presented in section 4. In section 5, both the analytical solutions are fitted with observed sunspots' growth and decay phases of the sunspots and conclusions from these results are presented.

2. Formulation of the equations

It is assumed that, in the convective envelope, fluid is incompressible. We also assume that the magnetic eddy diffusivity η with value represented by the appropriate average. Magnetic field $\mathbf{B}$ and the velocity field $\mathbf{V}$ vectors are expressed as

$$\mathbf{B} = P\hat{\mathrm{I}}_{\vartheta} + T\hat{\mathrm{I}}_{\varphi} \ , \tag{1}$$

$$\mathbf{V} = U\hat{\mathrm{I}}_{\vartheta} + r\Omega sin\theta\hat{\mathrm{I}}_{\varphi} \ , \tag{2}$$

where $\hat{\mathrm{I}}_{\vartheta}$ and $\hat{\mathrm{I}}_{\varphi}$ are the unit vectors along heliographic latitude ϑ and longitude φ of the sun; P, T, U and Ω are scalar functions. P and T are scalar functions that represent poloidal and toroidal parts of the the magnetic field structures and U and Ω are scalar functions that represent poloidal (meridional) and toroidal (angular velocity) parts of the velocity field structures. Equation of continuity is

$$\frac{\partial\rho}{\partial t} + \rho\nabla.\mathbf{V} = 0. \tag{3}$$

As the life spans ($\sim$ weeks to months) of sunspots are very much larger than the time scales ($\sim$ minutes) of ambient density perturbations in the convective envelope, we have $\frac{\partial\rho}{\partial t} = 0$ and the resulting equation is

$$\rho\nabla.\mathbf{V} = \mathbf{0} \tag{4}$$

where ρ is the density. Similarly as magnetic diffusivity is assumed to be constant, magnetic induction equation is

$$\frac{\partial\mathbf{B}}{\partial t} = curl(\mathbf{V}\ \times \mathbf{B}) + \eta\nabla^2\mathbf{B}. \tag{5}$$

This magnetic induction equation determines growth and decay of the sunspot. The first term on right hand side (RHS) is the source term that enhances the magnetic flux of the sunspot and second term on RHS is the sink term that attempts to destroy the generated magnetic flux. As magnetic induction equation in turn depends upon velocity and diffusivity η, these two source and sink terms are important and dictate the growth and decay of the sunspots. We solve the induction equation by considering the source and sink terms separately for the following reasons. In case of region of positive rotational gradient from base of convective envelope to $0.935R_{\odot}$, rate of increase of magnetic flux that mainly depends upon fluctuations of increase in rotational gradient is dominant compared to magnetic diffusivity. As for region of negative rotational gradient from $0.935R_{\odot}$ to $1.0R_{\odot}$, fluctuations in decreasing rotational gradient, increasing magnetic diffusivity (as magnetic diffusivity $\eta \sim T^{-3/2}$, where T is ambient temperature) and dominant radiational effects near the surface remove and destroy the magnetic flux.

3. Solution for growth of the sunspot

After substituting equations (1) and (2) in equation (5) and also by satisfying the continuity equation (4), by considering a source (first) term of the toroidal component of the induction equation in spherical coordinates is

$$\frac{\partial T}{\partial t} = (\frac{UTcot\theta}{r}) + (Psin\theta\frac{\partial \Omega}{\partial \theta} - \frac{U}{r}\frac{\partial T}{\partial \theta}) + (T\frac{\partial \Omega}{\partial \phi} - \Omega\frac{\partial T}{\partial \phi}) \ , \tag{6}$$

where r, θ and ϕ are radial, latitudinal and longitudinal variables in spherical coordinates. The last term in RHS of the above equation can be simplified further as follows

$$\Omega = \frac{\phi_2 - \phi_1}{t_2 - t_1} = \frac{\partial \phi}{\partial t} \ , \tag{7}$$

where Ω is the angular velocity, ϕ_1 and ϕ_2 are changes in longitudinal displacement from time t_1 and t_2 respectively. Hence we have following equations

$$\partial \phi = \Omega \partial t \ , \tag{8}$$

$$T\frac{\partial \Omega}{\partial \phi} = \frac{T}{\Omega}\frac{\partial \Omega}{\partial t} \ , \tag{9}$$

and

$$\frac{\partial T}{\partial \phi} = \frac{\partial T}{\partial t}\frac{\partial t}{\partial \phi} = \frac{\partial T}{\partial t}\frac{1}{\Omega}. \tag{10}$$

Hence,

$$(T\frac{\partial \Omega}{\partial \phi} - \Omega\frac{\partial T}{\partial \phi}) = (\frac{T}{\Omega}\frac{\partial \Omega}{\partial t} - \frac{\partial T}{\partial t}). \tag{11}$$

With these equations, equation (6) can be written as

$$2\frac{\partial T}{\partial t} = (\frac{UTcot\theta}{r}) + (Psin\theta\frac{\partial \Omega}{\partial \theta} - \frac{U}{r}\frac{\partial T}{\partial \theta}) + (\frac{T}{\Omega}\frac{\partial \Omega}{\partial t}). \tag{12}$$

Perturb this induction equation by taking the variables $\Omega = \Omega_0 + \Omega^{'}$, $T = T_0 + T^{'}$ and $= U_0 + U^{'}$ such that $\frac{\partial \Omega_0}{\partial t} = \frac{\partial T_0}{\partial t} = \frac{\partial U_0}{\partial t} = 0$ and magnitudes of fluctuating $\Omega^{'}$, $T^{'}$ and $^{'}$ components are assumed to be very small compared to steady parts Ω_0 and T_0. This condition also implies that magnitudes of products of the fluctuating components are nearly zero. Further it is assumed that poloidal component of the magnetic field structure P is constant and it's magnitude is very small compared to magnitude of toroidal magnetic field structure. This reasonable assumption is consistent with the observed strength of solar magnetic field structure that during 11 years period strength of poloidal field structure ($\sim$ 1 G) is << strength of toroidal magnetic field structure ($\sim 10^3$ G). That means the

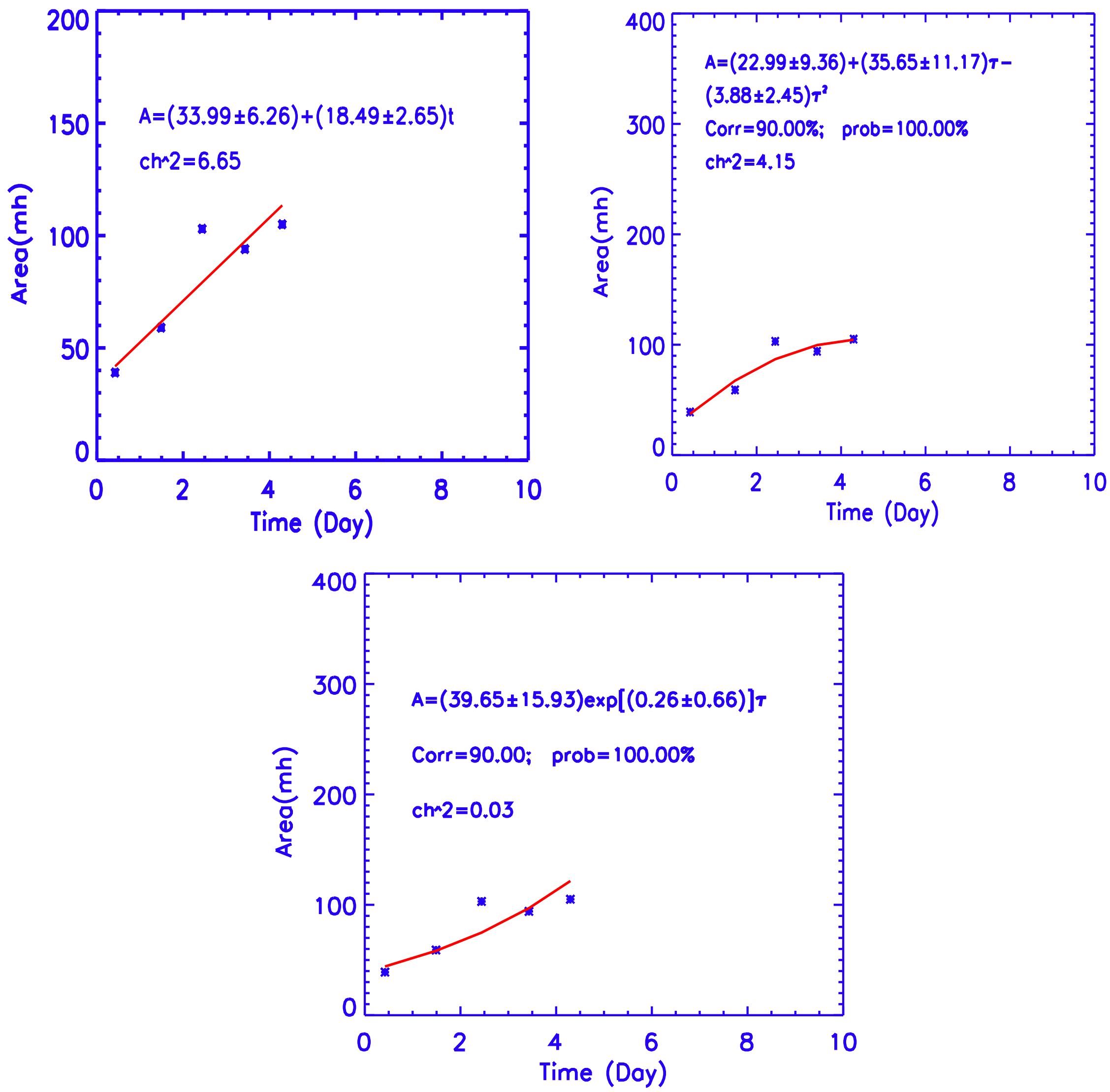

Fig. 2.— Evolution of growth of area A of non-recurrent sunspot group at a latitude region of 0 -10° that has lifespan of 8 days. Red line is theoretical area growth curve over plotted on the observed area growth curve (blue cross points).

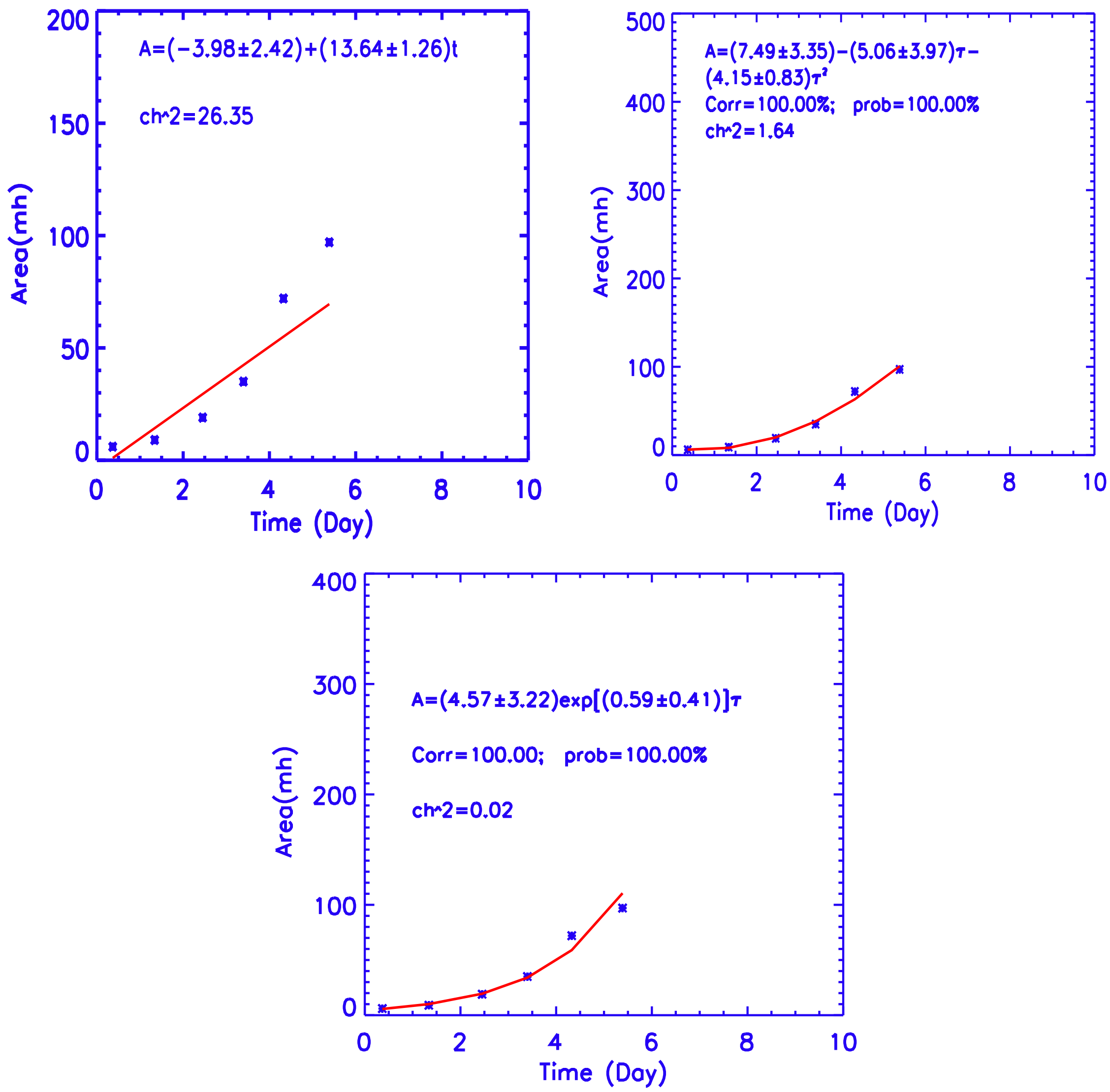

Fig. 3.— Evolution of growth of area A of non-recurrent sunspot group at a latitude region of 0 -10° that has lifespan of 8 days. Red line is theoretical area growth curve over plotted on the observed area growth curve (blue cross points).

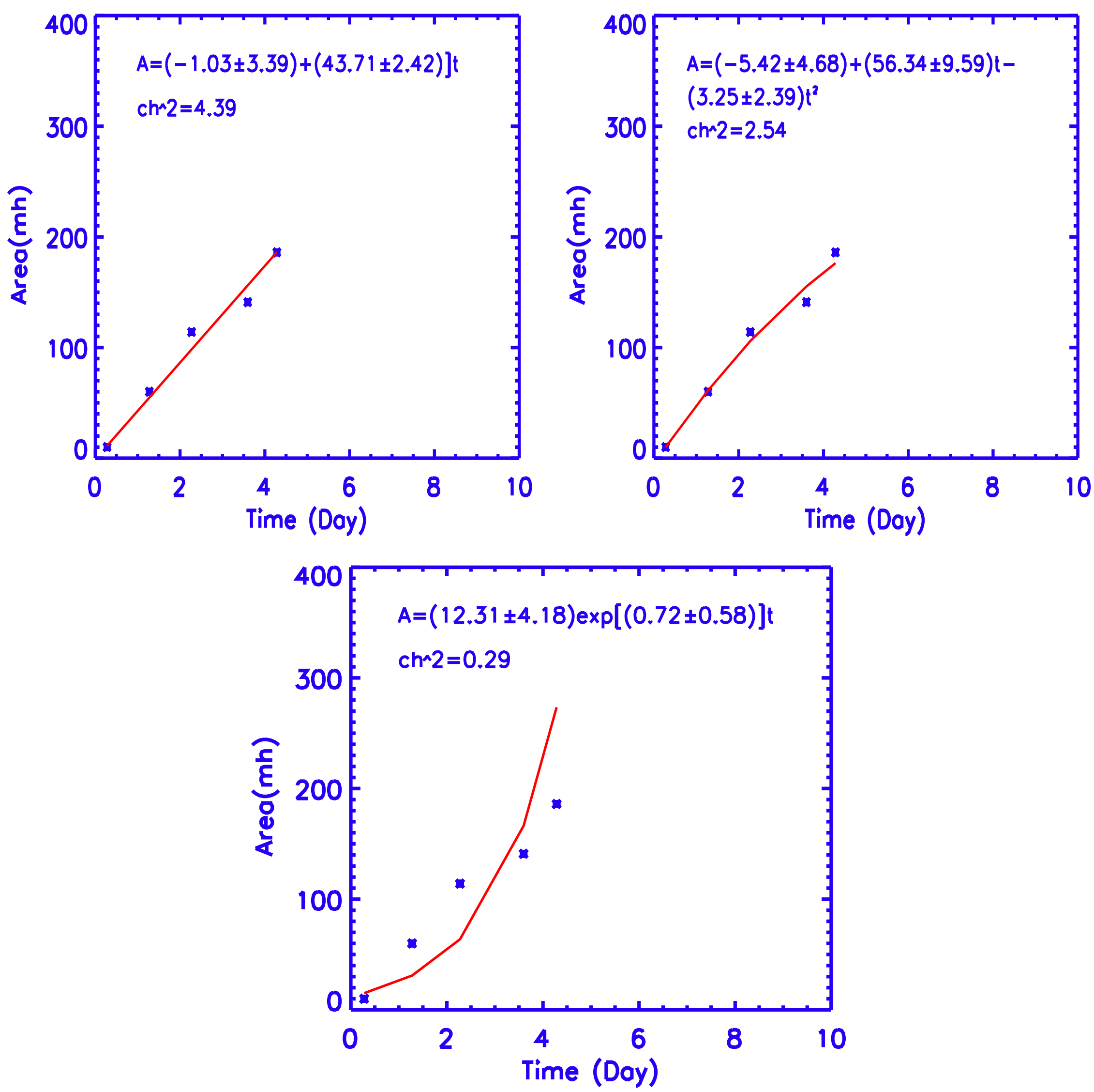

Fig. 4.— Evolution of growth of area A of non-recurrent sunspot group at a latitude region of 10 -20° that has lifespan of 8 days. Red line is theoretical area growth curve over plotted on the observed area growth curve (blue cross point).

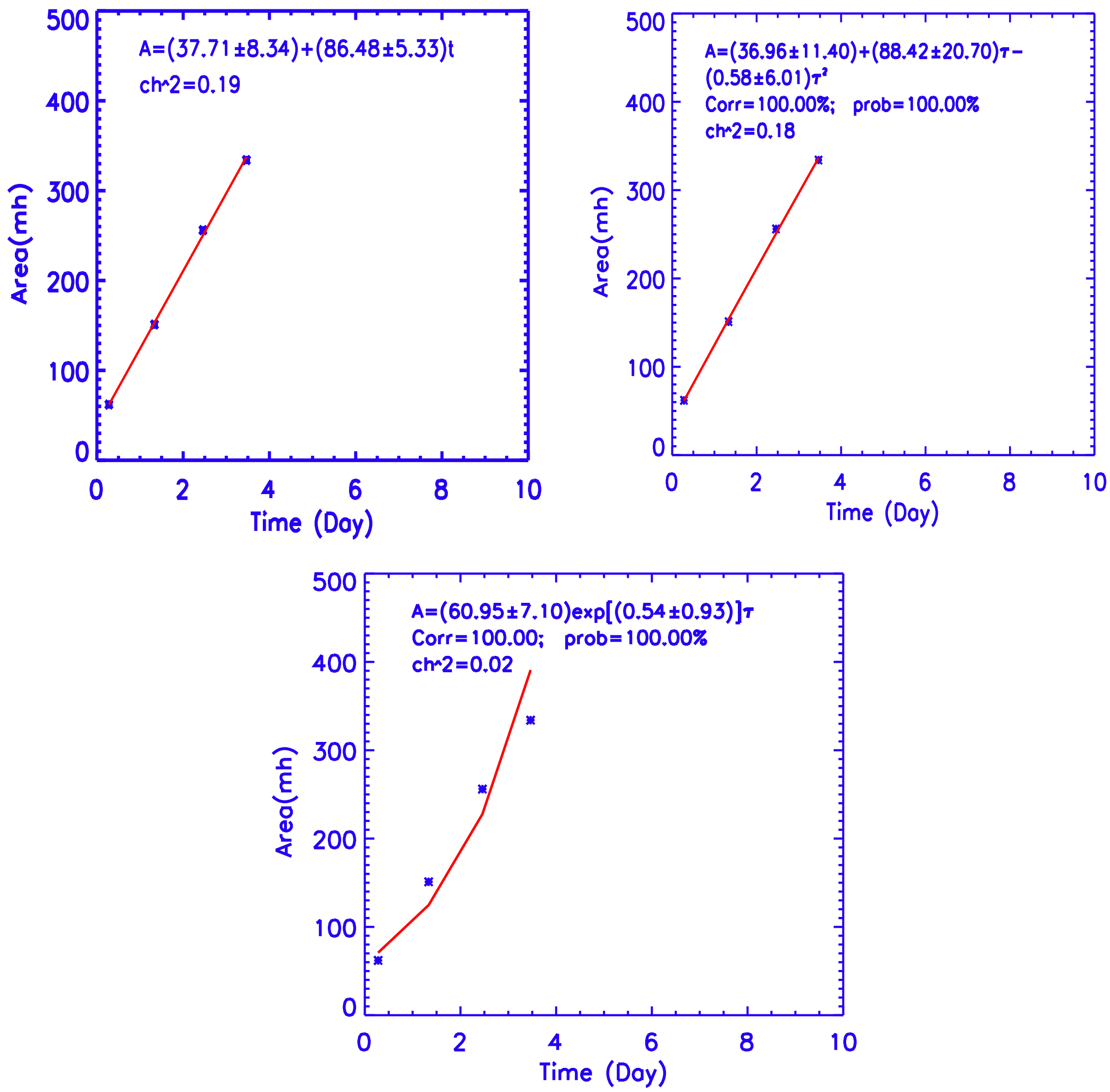

Fig. 5.— Evolution of growth of area A of non-recurrent sunspot group at a latitude region of 10 -20° that has lifespan of 10 days. Red line is theoretical area growth curve over plotted on the observed area growth curve (blue cross points).

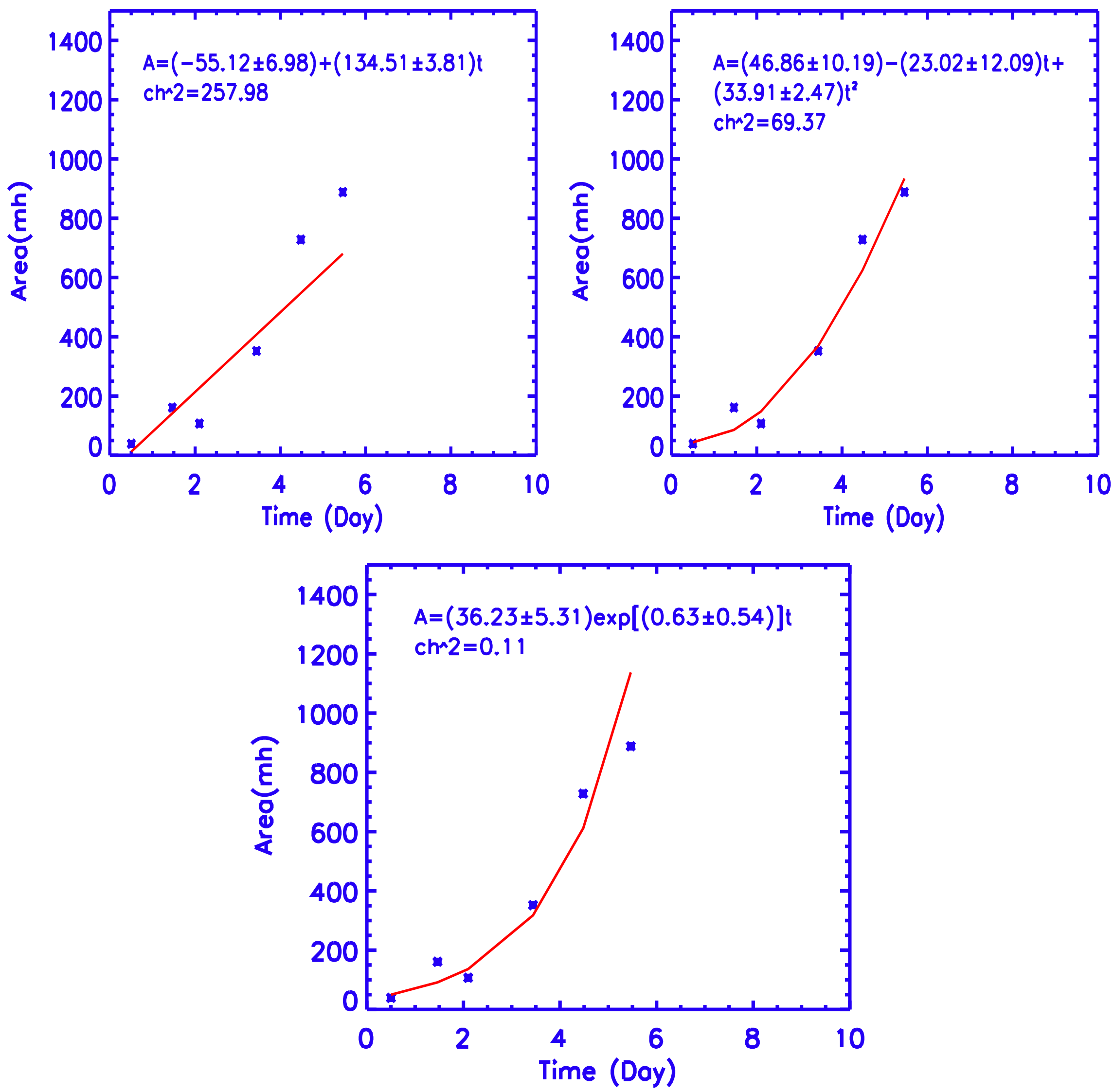

Fig. 6.— Evolution of growth of area A of non-recurrent sunspot group at a latitude region of 20 -30° that has lifespan of 10 days. Red line is theoretical area growth curve over plotted on the observed area growth curve (blue cross points).

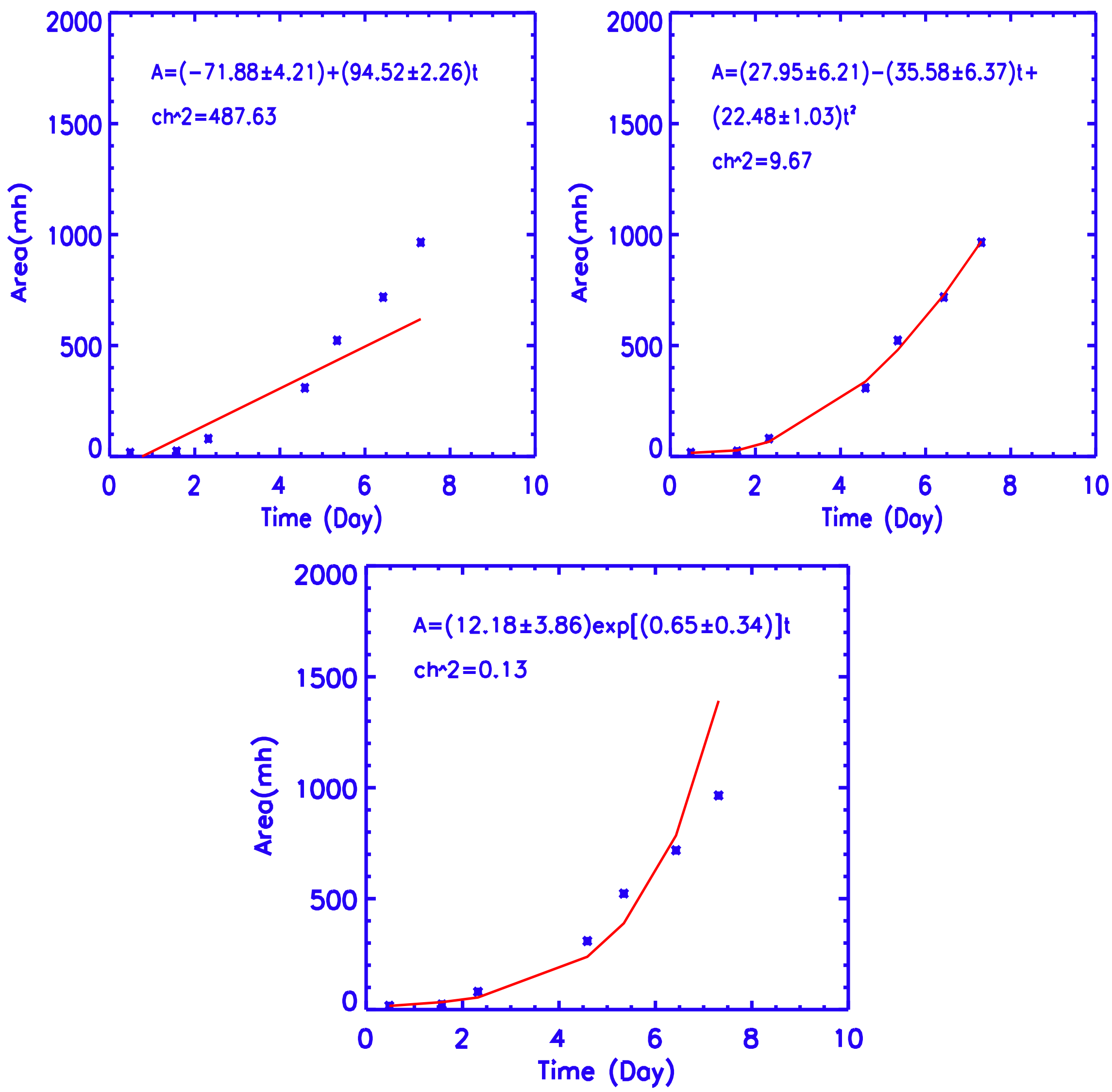

Fig. 7.— Evolution of growth of area A of non-recurrent sunspot group at a latitude region of 20 -30° that has lifespan of 10 days. Red line is theoretical area growth curve over plotted on the observed area growth curve (blue cross points).

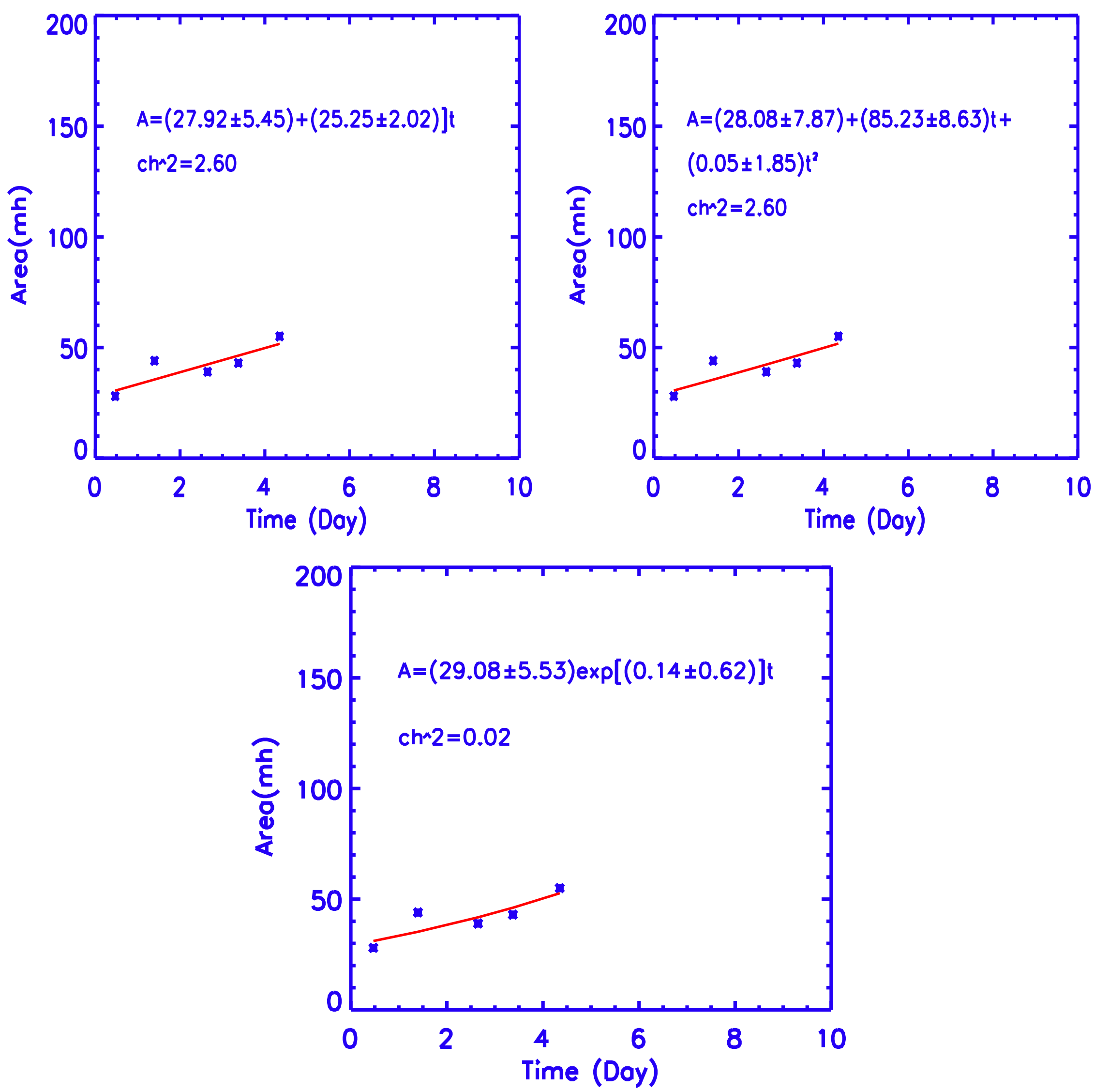

Fig. 8.— Evolution of growth of area A of non-recurrent sunspot group at a latitude region of 30 -40° that has lifespan of 8 days. Red line is theoretical area growth curve over plotted on the observed area growth curve (blue cross points).

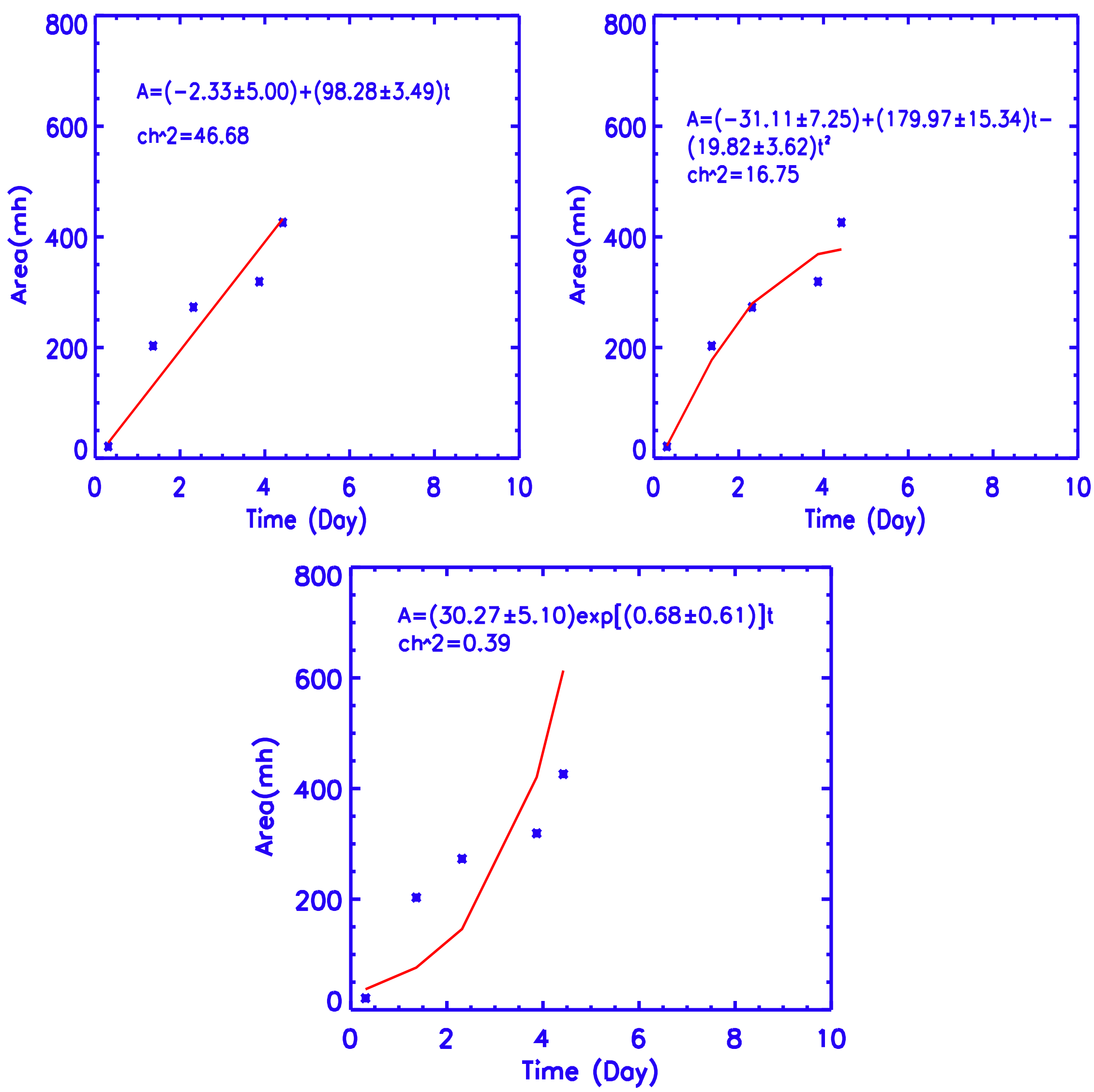

Fig. 9.— Evolution of growth of area A of non-recurrent sunspot group at a latitude region of 30 -40° that has lifespan of 9 days. Red line is theoretical area growth curve over plotted on the observed area growth curve (blue cross point).

fluctuating term $(Psin\theta\frac{\partial\Omega^{'}}{\partial\theta})$ is neglected. Hence resulting time dependent part of toroidal component of global magnetic field structure for the Alfven wave perturbations along the direction of rotation is given as follows

$$2\frac{\partial T^{'}}{\partial t} = (\frac{U_0 T^{'} + U^{'} T_0}{r})cot\theta - (\frac{U_0}{r}\frac{\partial T^{'}}{\partial\theta} + \frac{U^{'}}{r}\frac{\partial T_0}{\partial\theta}) + (\frac{T_0}{\Omega_0}\frac{\partial\Omega^{'}}{\partial t}). \tag{13}$$

Derivative $\frac{\partial T^{'}}{\partial\theta}$ can be modified as follows

$$\frac{\partial T^{'}}{\partial\theta} = \frac{\partial T^{'}}{\partial t}\frac{\partial t}{\partial\theta} = \frac{\partial T^{'}}{\partial t}\frac{1}{U_0} \ , \tag{14}$$

where U_0 is steady part of meridional circulation.

On both sides of the equation (13), multiply the term $A = \pi S^2$ of the flux tube (where S is radius of the tube at a particular depth) and resulting equation for rate of change of magnetic flux or area (as area of the sunspot is directly proportional to magnetic flux) of the sunspot is given as follows

$$\frac{\partial A}{\partial t} = \frac{U_0 cot\theta A}{2r+1} + \frac{S^2 U^{'}}{2r+1}(T_0 cot\theta - \frac{\partial T_0}{\partial\theta}) + \frac{rS^2 T_0}{(2r+1)\Omega_0}\frac{\partial\Omega^{'}}{\partial t}. \tag{15}$$

This equation suggests that rate of change of area of the sunspot is a function of steady parts of poloidal and toroidal velocity field structures, radial variations in fluctuations of the meridional velocity and steady part of toroidal component of magnetic field structure respectively. Although momentum equation is necessary (for the hydrostatic equilibrium of internal structure of the sun, as $\frac{\partial\Omega^{'}}{\partial t}$ is proportional to fluctuating parts of advective terms, Lorentzian force and variation in the second derivative of angular velocity), as fluctuating terms are assumed to be small (although in principle not to be neglected), radial variation of last two terms in RHS is neglected. Hence, with the initial conditions that at time $t = 0$, area $A = A_0$ (initial area), solution yields the following relationship between increase of flux tube area (while it raises in the positive rotational gradient) with respect to time.

$$A(t) = A_0 e^{\frac{(U_0 cot\theta)t}{2r+1}} \tag{16}$$

As for Alfven wave perturbations that are opposite to the direction of angular velocity, solution for growth of the sunspot is

$$A(t) = A_0 e^{\frac{(-U_0 cot\theta)t}{2r+1}} \ . \tag{17}$$

Hence, in the region of positive rotational gradient, simultaneous growth and decay of the Alfven wave perturbations exist yielding net exponential growth of the sunspot. Another

interesting property of solution (equation 16) is that exponent of the growth part depends upon magnitude of meridional velocity U_0, $cot\theta$ and the depth of the foot point of the flux tube where it is anchored. It is not known how the meridional velocity varies with depth and it is assumed to be constant. Thus as time progresses, due to buoyancy, anchored feet lifts from interior in the positive rotational gradient (until it reaches maximum angular velocity at the depth $0.935R_\odot$) with an exponential growth of area of the sunspot.

If one keeps the ratio $\frac{U_0}{2r+1}$ constant at a particular depth (say near the surface), exponent of solution for growth of area is directly proportional to $cot\theta$. That means by the property of $cot\theta$ function, spots at the lower co-latitudes θ (or higher heliographic latitudes) grow very fast compared to the spots that grow at the lower co-latitudes (or lower heliographic latitudes, i,e., near the equator). This important property of sunspot's growth will be tested in the following sections.

Once sunspot's anchoring feet enters the negative rotational gradient, the picture will be different and it will be known from the next section that area of the sunspot decays exponentially and ultimately disappears on the surface.

4. Solution for decay of the sunspot

After substituting equations (1) and (2) in equation (5) and also by satisfying the continuity equation (4), resulting toroidal component of the induction equation with a sink term alone in spherical coordinates is

$$\frac{\partial T}{\partial t} = \eta[\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{1}{r^2 sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{cot\theta}{r^2}\frac{\partial T}{\partial \theta} - \frac{T}{r^2 sin^2\theta}]. \tag{18}$$

Adopting a similar method in the previous section, this equation can be transformed into following equation for steady part of toroidal component of the induction equation

$$\frac{\partial^2 T}{\partial t^2} - (r^2 sin^2\theta\frac{\partial \Omega}{\partial t} + \frac{\Omega^2 r^2 sin^2\theta}{\eta})\frac{\partial T}{\partial t} + \frac{\Omega sin^2\theta}{\eta}\frac{\partial^2 T}{\partial \theta^2} + \frac{\Omega sin^2\theta cot\theta}{\eta}\frac{\partial T}{\partial \theta} - \frac{\eta T}{r^2 sin^2\theta} = 0. \tag{19}$$

Following similar perturbation method in the previous section, we get the following time dependent component of the toroidal component of magnetic field structure

$$A_1\frac{\partial^2 T^{'}}{\partial t^2} + A_2\frac{\partial T^{'}}{\partial t} + A_3 T^{'} = 0\ , \tag{20}$$

where

$$A_1 = 1 + \frac{\Omega^{'} sin^2\theta}{\eta {U_0}^2}\ , \tag{21}$$

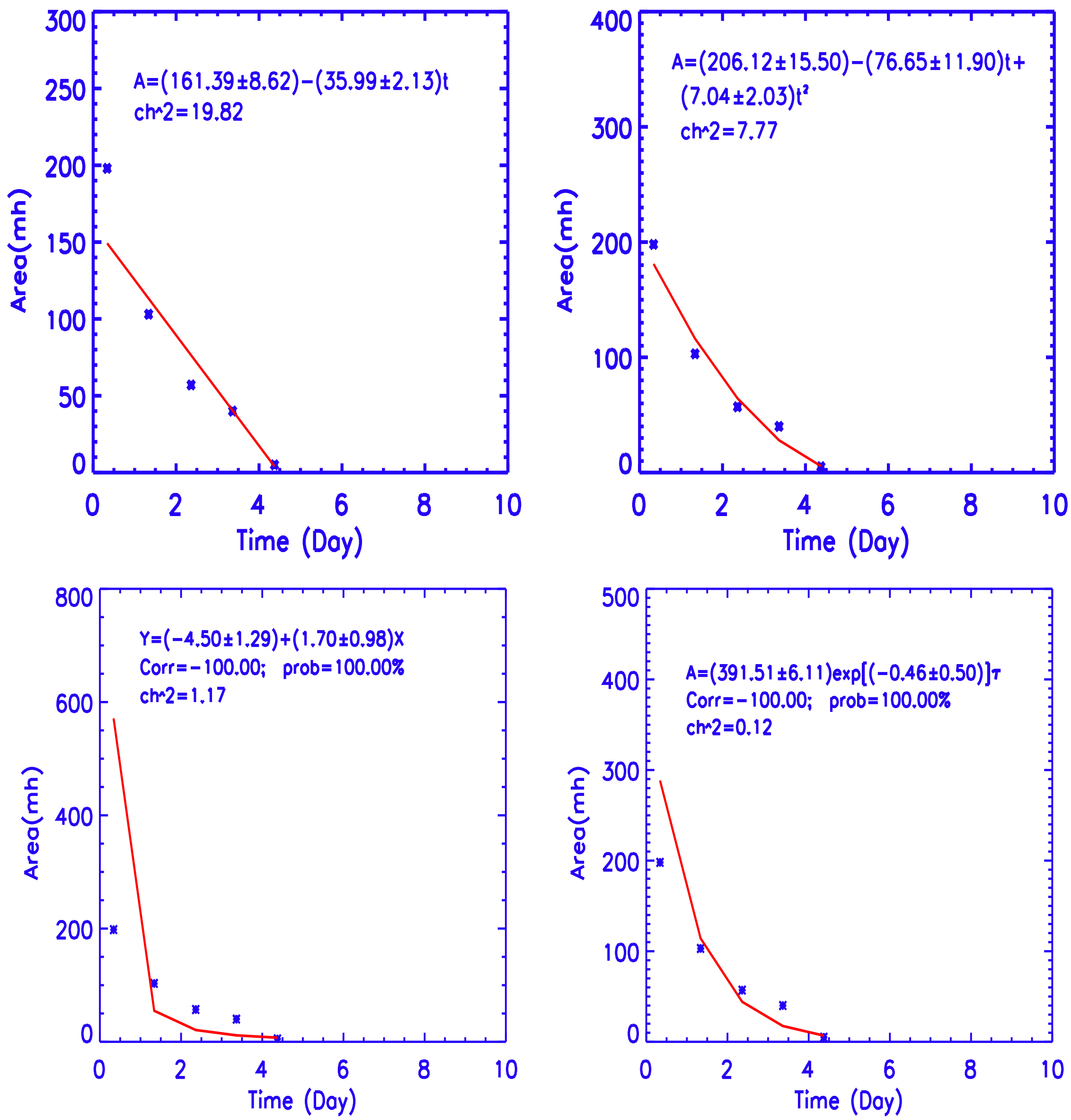

Fig. 10.— Evolution of decay of area A of non-recurrent sunspot group at a latitude region of 0 -10° that has lifespan of 9 days. In Fig (c), X=ln(Time) and Y=-ln(A). Red line is theoretical area decay curve over plotted on the observed area decay curve (blue cross points).

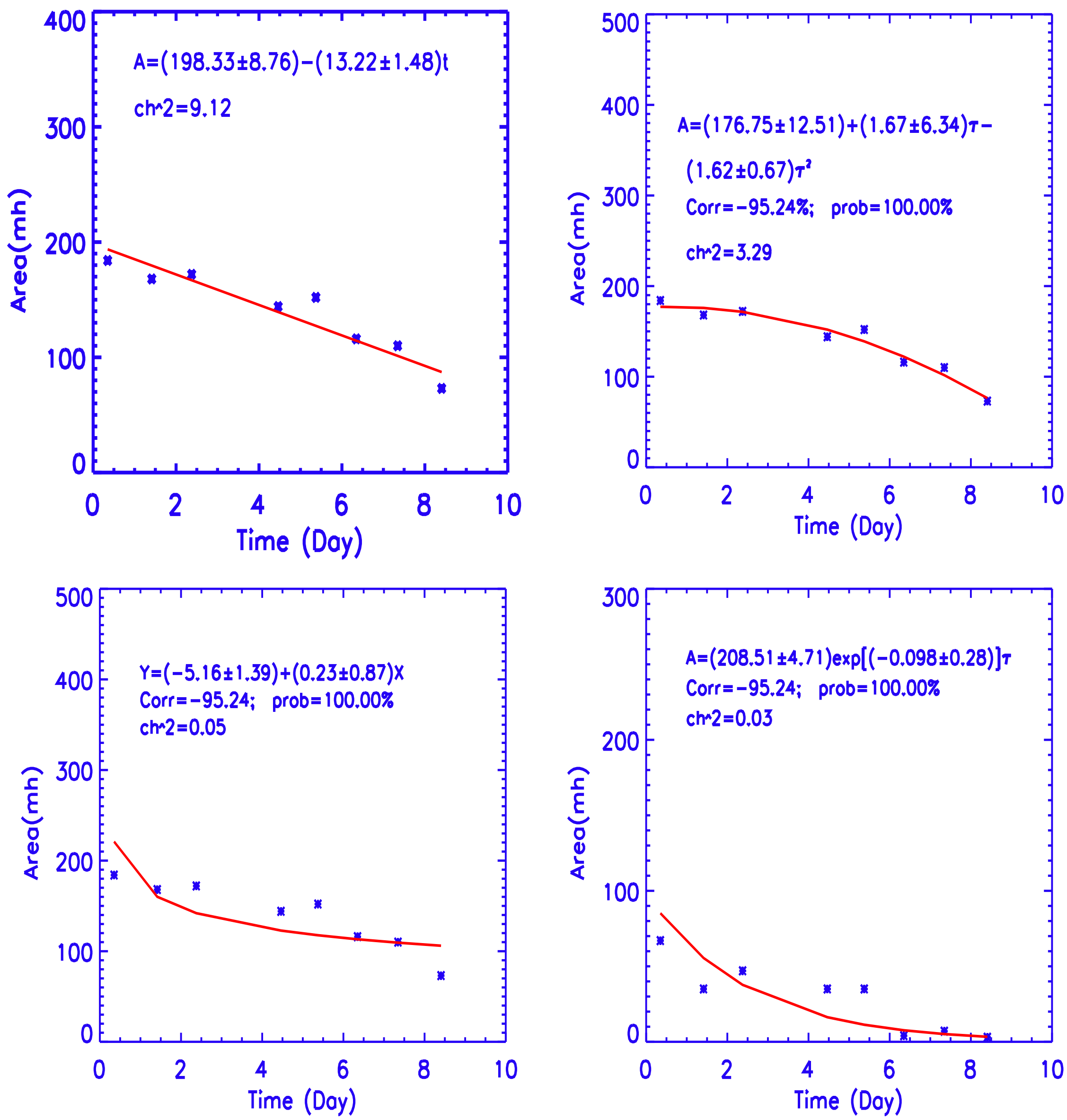

Fig. 11.— Evolution of decay of area A of non-recurrent sunspot group at a latitude region of 0 -10° that has lifespan of 9 days. In Fig (c), X=ln(Time) and Y=-ln(A). Red line is theoretical area decay curve over plotted on the observed area decay curve (blue cross points).

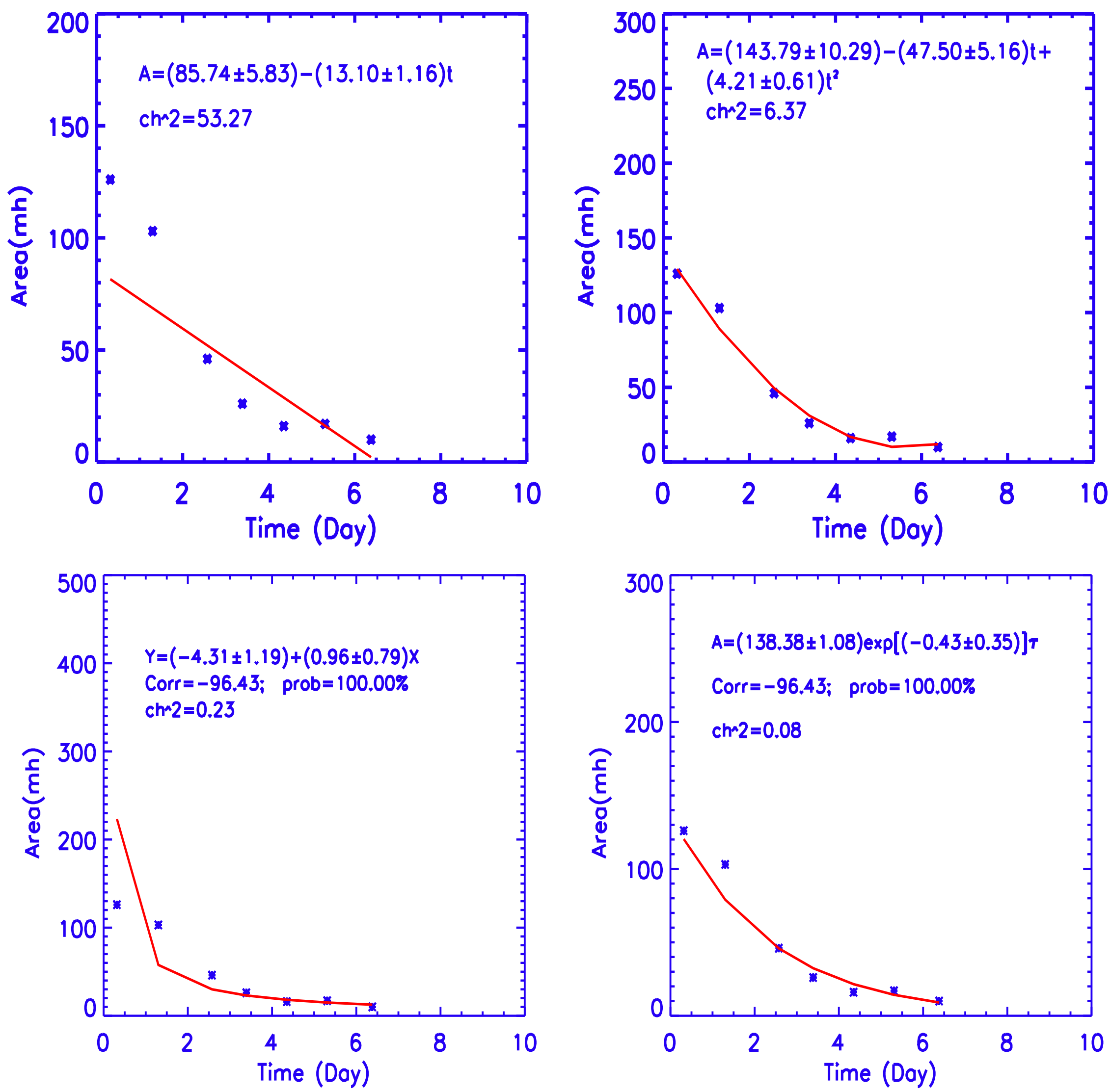

Fig. 12.— Evolution of decay of area A of non-recurrent sunspot group at a latitude region of 10 -20° that has lifespan of 10 days. In Fig (c), X=ln(Time) and Y=-ln(A). Red line is theoretical area decay curve over plotted on the observed area decay curve (blue cross points).

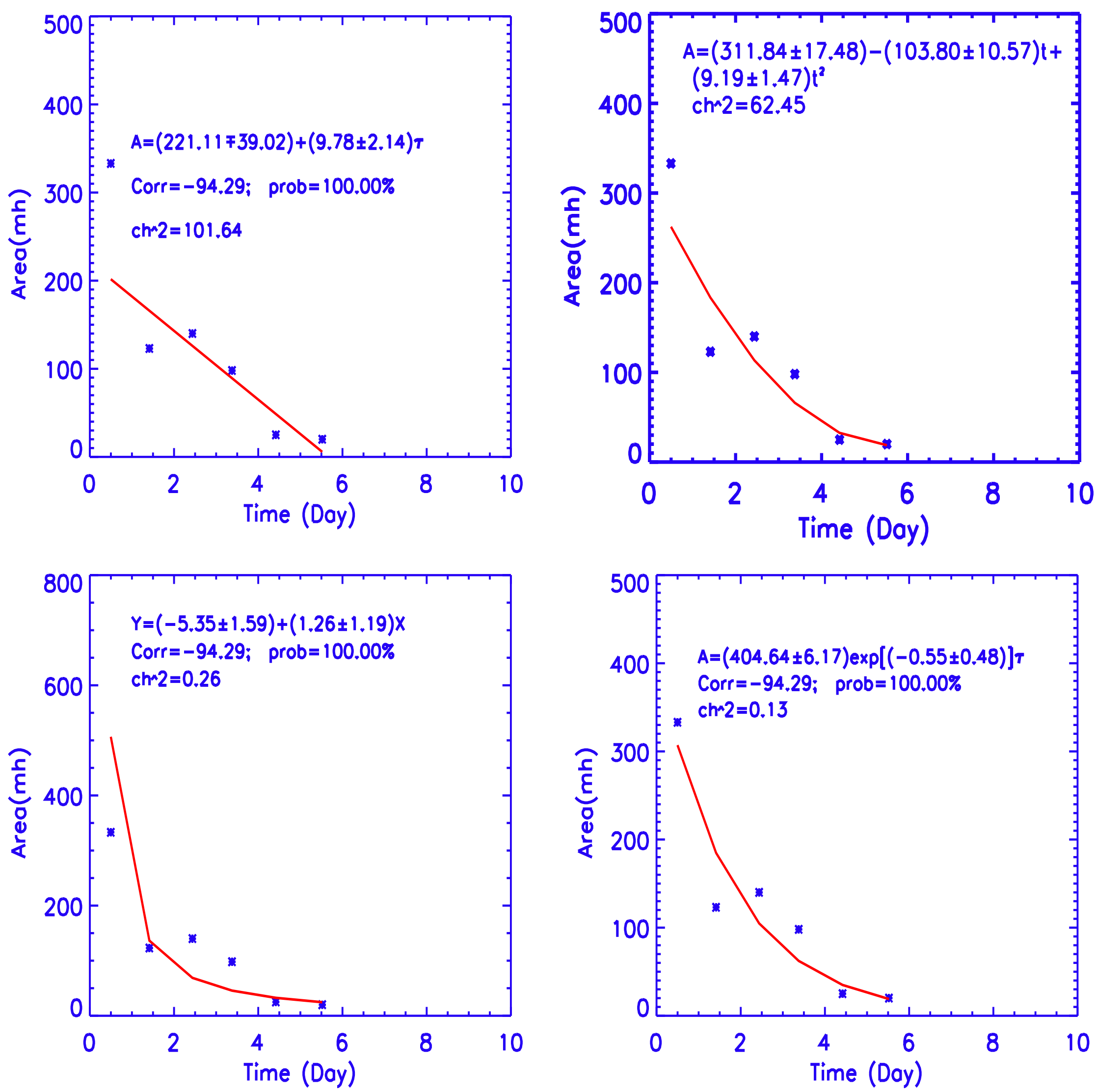

Fig. 13.— Evolution of decay of area A of non-recurrent sunspot group at a latitude region of 10 -20° that has lifespan of 9 days. In Fig (c), X=ln(Time) and Y=-ln(A). Red line is theoretical area decay curve over plotted on the observed area decay curve (blue cross points).

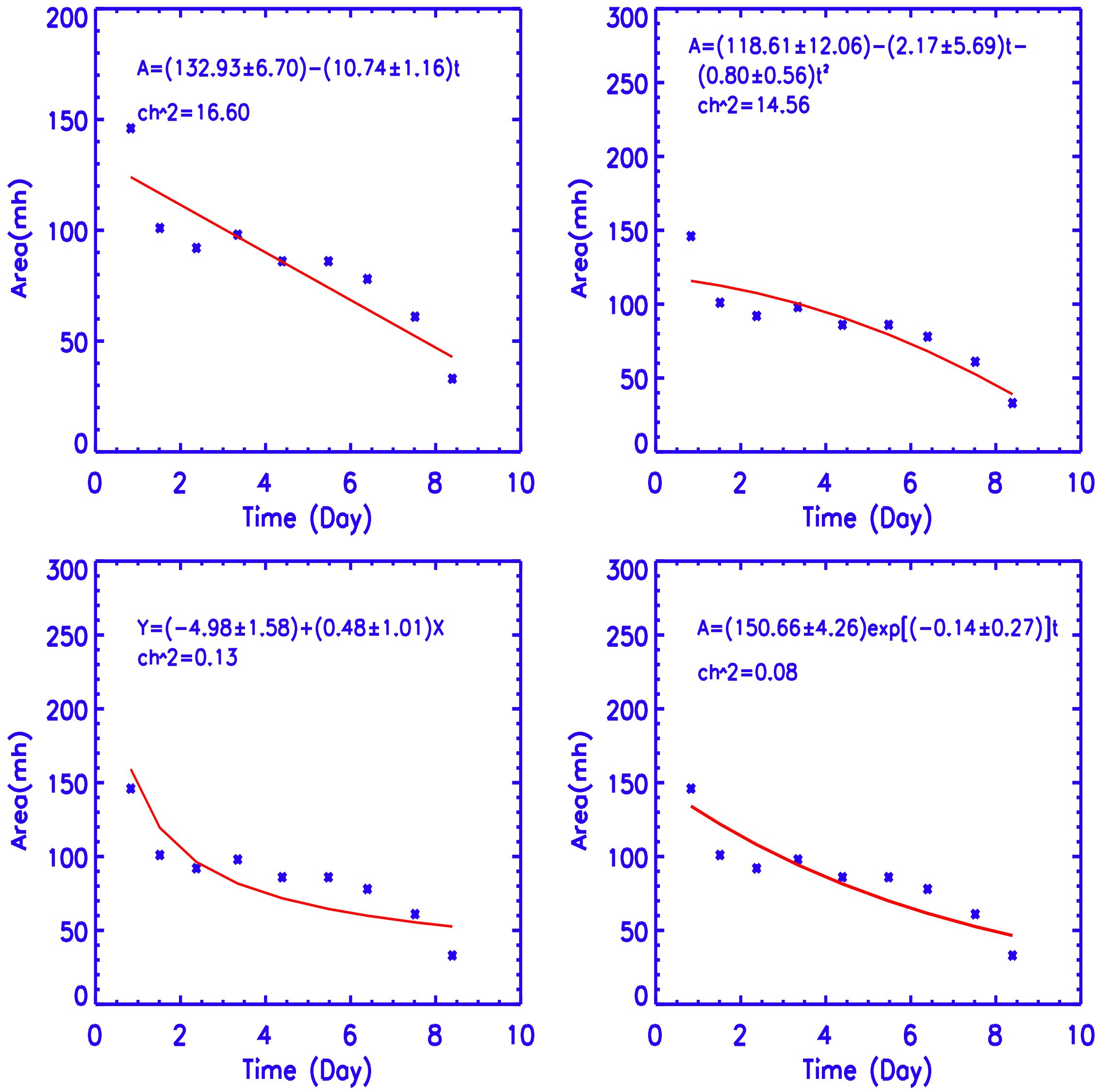

Fig. 14.— Evolution of decay of area A of non-recurrent sunspot group at a latitude region of 20 -30° that has lifespan of 10 days. In Fig (c), X=ln(Time) and Y=-ln(A). Red line is theoretical area decay curve over plotted on the observed area decay curve (blue cross points).

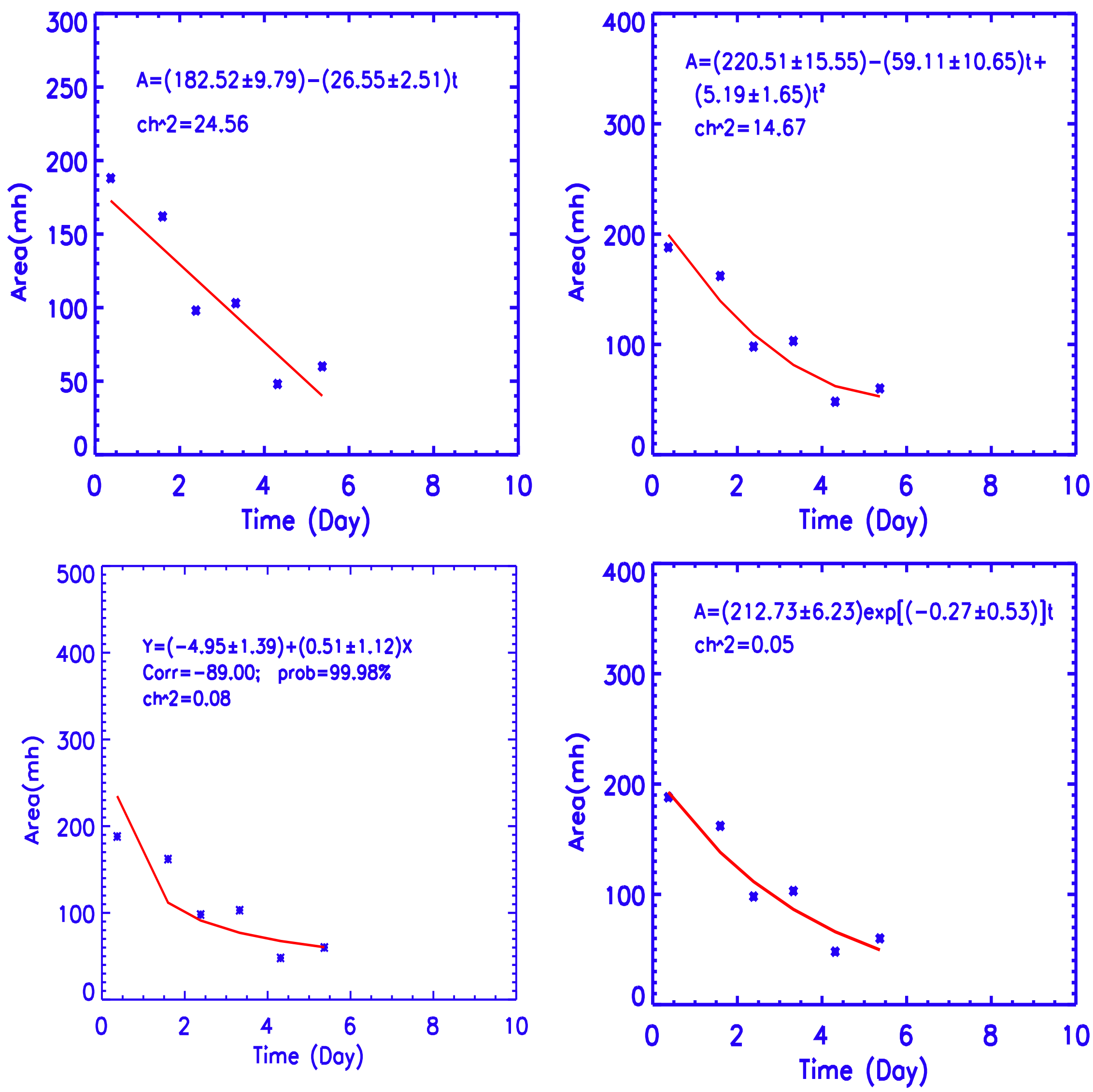

Fig. 15.— Evolution of decay of area A of non-recurrent sunspot group at a latitude region of 20 -30° that has lifespan of 10 days. In Fig (c), X=ln(Time) and Y=-ln(A). Red line is theoretical area decay curve over plotted on the observed area decay curve (blue cross points).

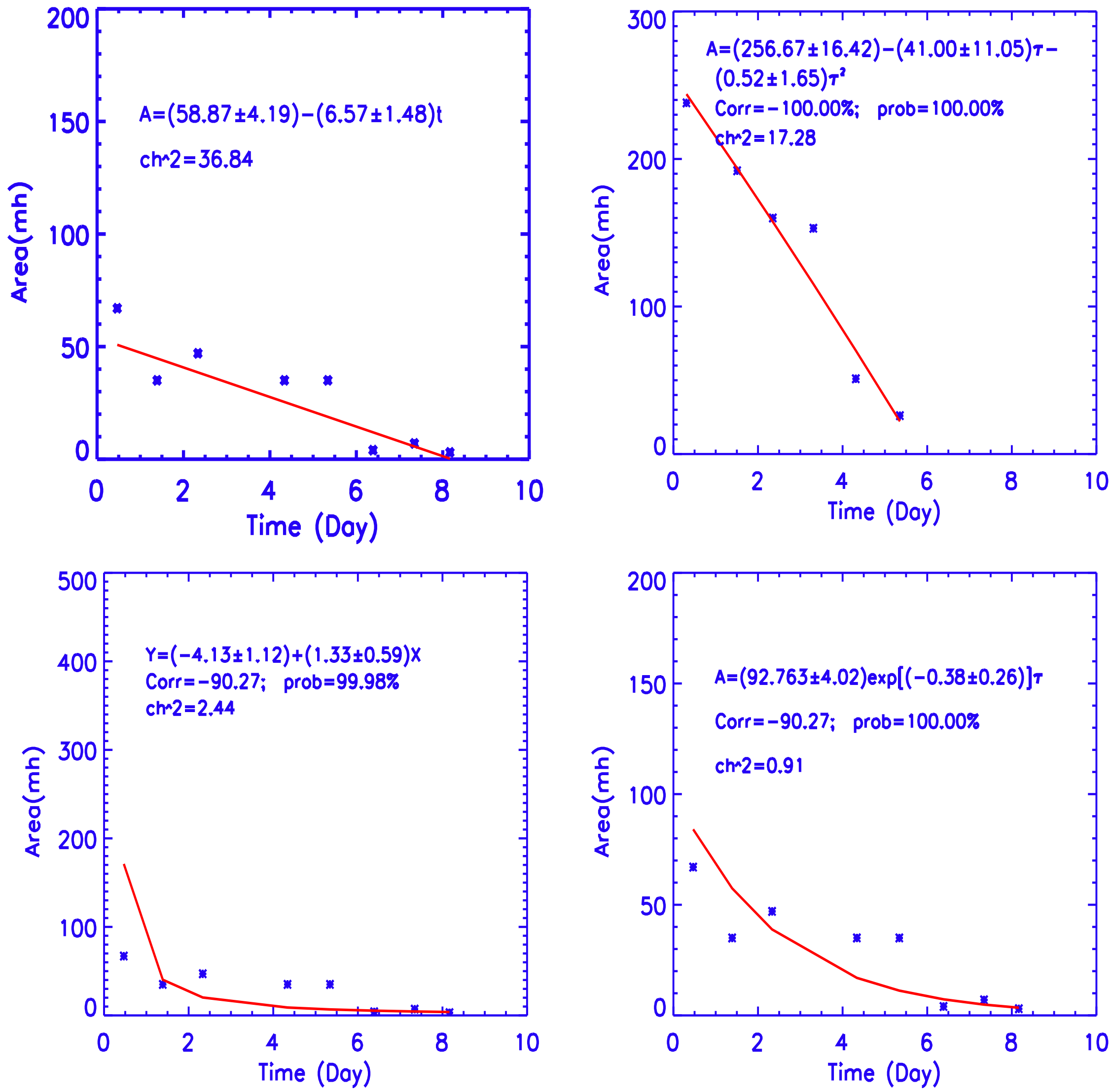

Fig. 16.— Evolution of decay of area A of non-recurrent sunspot group at a latitude region of 30 -40° that has lifespan of 8 days. In Fig (c), X=ln(Time) and Y=-ln(A). Red line is theoretical area decay curve over plotted on the observed area decay curve (blue cross points).

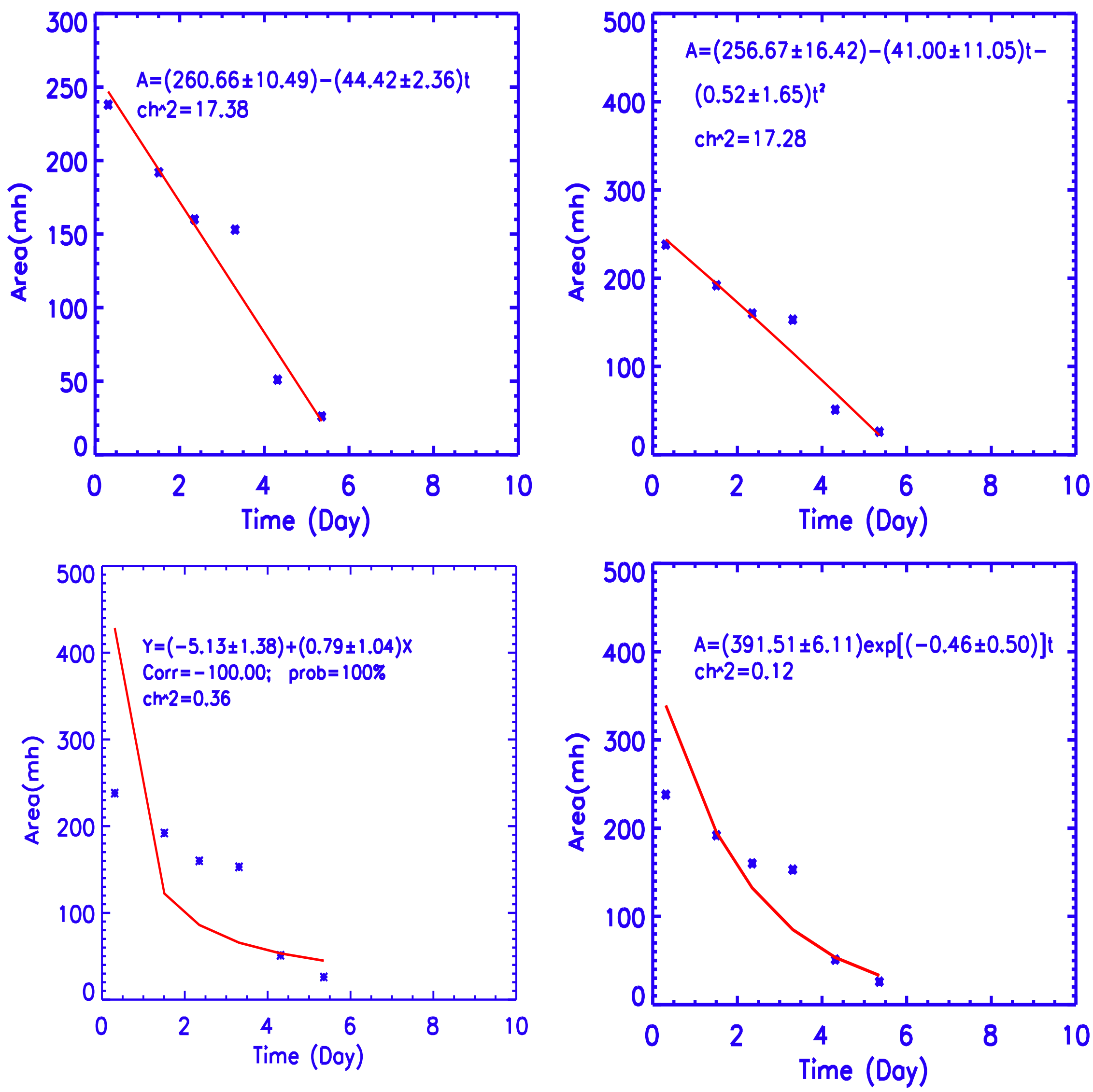

Fig. 17.— Evolution of decay of area A of non-recurrent sunspot group at a latitude region of 30 -40° that has lifespan of 8 days. In Fig (c), X=ln(Time) and Y=-ln(A). Red line is theoretical area decay curve over plotted on the observed area decay curve (blue cross points).

$$A_2 = r^2 sin^2\theta \frac{\partial \Omega^{'}}{\partial t} + \frac{\Omega_0^2 r^2 sin^2\theta}{\eta} + 2\frac{\Omega_0 \Omega^{'} r^2 sin^2\theta}{\eta} + \frac{\Omega^{'} sin^2\theta}{\eta {U_0}^3}\frac{\partial U_0}{\partial t} - \frac{\Omega^{'} sin^2\theta cot\theta}{\eta U_0} \ , \tag{22}$$

and

$$A_3 = -\Omega_0. \tag{23}$$

On both sides of the equation (20), multiply a term $A = \pi S^2$ of the flux tube (where S is radius of the tube). As amplitude $\Omega^{'}$ of fluctuations in angular velocity assumed to be negligible compared to steady part of the angular velocity, then we have the analytical solution for variation of the sunspot area with respect to time in case of perturbations along the direction of angular velocity

$$A(t) = c_1 e^{D_1 t} + c_2 e^{-D_2 t} \ , \tag{24}$$

where c_1 and c_2 are integrational constants and,

$$D_1 = \frac{1}{2}[\frac{\Omega_0^2 r^2 sin^2\theta}{\eta} + \sqrt{(\frac{\Omega_0^2 r^2 sin^2\theta}{\eta})^2 + 4\Omega_0}] \ , \tag{25}$$

and

$$D_2 = \frac{1}{2}[\frac{\Omega_0^2 r^2 sin^2\theta}{\eta} - \sqrt{(\frac{\Omega_0^2 r^2 sin^2\theta}{\eta})^2 + 4\Omega_0}] \ , \tag{26}$$

As second term in the square root is negligible compared to the first term, solution for decay part of the area of the sunspot with respect to time is given as

$$A(t) = c_1 e^{(\frac{\Omega_0^2 {R_\odot}^2 x^2 sin^2\theta}{\eta})t} + c_2 \ , \tag{27}$$

where $x = \frac{r}{R_\odot}$ and $R_\odot$ is the radius of the sun. Although, mathematically, solution has an exponential growth, physically, in the region of negative rotational gradient as flux tube lifts its anchored feet (due to buoyancy) toward surface, the distance (difference between sunspot's anchored depth and the surface) r decreases, angular velocity Ω_0 decreases and magnetic diffusivity η ($T^{-3/2}$, where T is ambient temperature) increases and hence resulting area decreases.

If one keeps the ratio Ω_0^2/η constant at a particular depth (say near the surface), exponent of the decay is directly proportional to $sin^2\theta$. That means spots at the lower co-latitudes θ (or higher heliographic latitudes) decay very slow compared to the spots that decay at the higher co-latitudes (or lower heliographic latitudes, i,e., near the equator). This important decay property will be tested in the following sections.

As for the Alfven wave perturbations opposite to the direction of angular velocity in the solar interior, solution yields

$$A(t) = c_1 e^{(\frac{-\Omega_0^2 {R_\odot}^2 x^2 sin^2\theta}{\eta})} t + c_2 \tag{28}$$

Hence, in the region of negative rotational gradient, at a particular latitude and depth, summation of these two solutions effectively constitutes the decay of area of the sunspots.

5. Result and Conclusions

In order to test results of the physical ideas on the growth and decay of the sunspots that are presented in the previous sections, data of time evolution of corrected areas of non-recurrent sunspot groups from Greenwich Photoheliographic Results (GPR) are used. For the four latitude zones of $0-10$, $10-20$, $20-30$ and $30-40$ degrees two spot groups that lie between $\pm$ 70 degree from the central meridian and life spans in the range of 8-10 days are considered.

In Figures 2-9, time evolution of growth of area of the non-recurrent sunspot groups are presented. It is assumed that sunspot area grows linearly, quadratically and exponentially and relevant laws are fitted with the observed growth of area of the sunspot groups. As measured uncertainties in the areas of sunspot groups are not available in GPR, it is assumed that growth and decay of area curves follow the Poisson distribution and hence uncertainty in each of measured area $A(t)$ (where t day of observation) is taken as $[A(t)]^{1/2}$. By knowing area $A(t)$ values and their uncertainties, all the three laws are fitted to the observed sunspots' area growth curves and are over plotted on top of the each plot. In all the Figures 2-9, the plots in the top are for the linear and quadratic fits and the the plot at the bottom is fit for the exponential growth law.

Similarly, in Figures 10-17, observed decay of area of the sunspot groups for all the four latitude zones are presented. In addition to three (viz., linear, quadratic, exponential) decay laws, a law of log-normal distribution is also considered for fitting the observed decay curves. In all the Figures 10-17, first and second plots in the top row are for linear and quadratic decay fits respectively. In the second row of Figures 10-17, log-normal and exponential decay fits are presented.

As for growth of the sunspots, it is interesting to note that among all the Figures 2-9, exponential fit is best one. This is also clearly evident from the χ^2 values presented in Table 1. In the Table 1, first column represents latitude of occurrence of the sunspot, second column represents lifespan and, columns 3-5 represent χ^2 values for linear, quadratic and exponential fits. It is to be noted that low value of χ^2 means, observed and expected curves are almost similar. In Table 2, constants C_1 and C_2 of exponential growth and decay parts of the area curve are presented. First column represents latitude of occurrence of the spot group, second and third columns represent the constants C_1 and C_2 that are determined

from the exponential growth and, fourth and fifth columns represent the constants that are determined from fits of exponential decay of the sunspot area curve respectively. Another important property, according to theoretical expectations presented in section 3 regarding growth of the sunspot, as is evident from Table 2 (see the third column) that the exponent of the exponential fit for the high heliographic latitude is high compared to the exponential fits for the low heliographic latitudes. That means the spots that formed at the high latitudes grow fast (with exponential growth) and the spots that are formed near the low heliographic latitudes grow slowly. As for decay of the sunspots, even though log-normal fit appears to be a very good fit among all the Figures 10-17, from criterion of goodness of fit of χ^2 value, exponential decay fit is best one. In fact this result is also clearly evident from the values of χ^2 presented in Table 3. Similarly we have another important property from these results. According to theoretical expectations presented in section 4 regarding decay of the sunspots, exponent of the exponential fit (see the 5^{th} column of Table 2) for the high heliographic latitude is very low compared to exponent of the exponential fits for the low heliographic latitudes. That means the spots that are formed at the high latitudes decay slowly compared to the spots that are formed near the low heliographic latitudes.

Even with approximations by neglecting fluctuations in poloidal (meridional) and toroidal (angular) components of velocity fields, theoretical solutions of growth (equation 16) and decay (equation 28) parts of sunspot's area evolutionary phases match with the observed area evolutionary phases. In order to understand a unique single solution for understanding growth and area decay curve, one should solve consistently full set of MHD equations (as the neglected fluctuations $\frac{\partial \Omega'}{\partial t}$ in turn depend upon fluctuations in the momentum equations).

From the observed characteristics of growth and decay of the sunspots at different latitudes on the surface and from the theoretical ideas presented in this study, one can safely conclude that sunspots are formed due to constructive interference of toroidal Alfven wave perturbations and, after attaining a critical strength in the convective envelope, due to buoyancy, sunspots raise along isorotational contours and reach the respective latitudes. It is understood from this study that growth and decay phases of the sunspots not only depend upon the surface physical characteristics , as this problem (especially decay part) was treated by the earlier studies, but also evolutionary history of internal dynamics and magnetic field structure of the sunspots while they raise toward the surface. As the sunspot is a three dimensional structure whose evolutionary history not only depends upon its internal structure but also on the ambient dynamic properties of the solar convective envelope that ultimately yields a combined solution of growth and decay of the sunspot.

Table 1: χ^2 fit for the laws of linear, quadratic and exponential growth of the sunspot.

Latitude	Life span (Days)	Linear	Quadratic	Exponential
0 - 10°	9	6.65	4.15	0.03
0 - 10°	9	26.35	1.64	0.02
10 - 20°	9	4.39	2.54	0.29
10 - 20°	10	0.19	0.18	0.02
20 - 30°	10	257.98	69.37	0.11
20 - 30°	10	487.63	9.67	0.13
30 - 40°	8	2.60	2.60	0.02
30 - 40°	9	46.68	16.75	0.39

Table 2: Values of constants obtained from growth and decay of the exponential fits.

	Growth		Decay	
Latitude	C_1	C_2	C_1	C_2
0 - 10°	39.65±15.93	0.26±0.66	395.44±6.17	0.93±0.56
0 - 10°	4.57±3.22	0.59±0.41	208.51±4.71	0.98±0.28
10 - 20°	12.31±4.18	0.72±0.58	138.38±1.08	0.43±0.35
10 - 20°	60.95±7.10	0.54±0.93	403.434±6.17	0.55±0.48
20 - 30°	36.23±5.31	0.63±0.54	5.02±1.45	0.14±0.27
20 - 30°	12.18±3.86	0.65±0.34	212.73±6.23	0.27±0.53
30 - 40°	29.08±5.53	0.14±0.62	92.76±4.02	0.38±0.26
30 - 40°	30.27±5.10	0.68±0.61	391.51±6.11	0.46±0.50

Table 3: χ^2 fit for the laws of linear, quadratic, log-normal and exponential decay of the sunspot.

Latitude	Life span (Days)	Linear	Quadratic	Log-normal	Exponential
0 - 10°	8	19.82	7.77	1.17	0.29
0 - 10°	9	9.12	3.29	0.05	0.03
10 - 20°	10	53.27	6.37	0.23	0.08
10 - 20°	8	101.64	62.45	0.26	0.13
20 - 30°	10	16.60	17.28	0.13	0.08
20 - 30°	10	24.56	3.29	0.08	0.05
30 - 40°	8	17.38	17.28	2.44	0.91
30 - 40°	8	36.84	31.51	0.36	0.12

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